# 超导简介

Introduction to Superconductivity

## Chapter 4 The Critical Magnetic Field

# Chapter 4 The Critical Magnetic Field

## Magnetism

$$d\mathbf{F} = id\mathbf{l} \times \mathbf{B}$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi r^2} id\mathbf{l} \times \hat{\mathbf{r}}$$

**Biot-Savart Law**

## Electrostatics

$$d\mathbf{F} = dq \cdot \mathbf{E}$$

$$d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

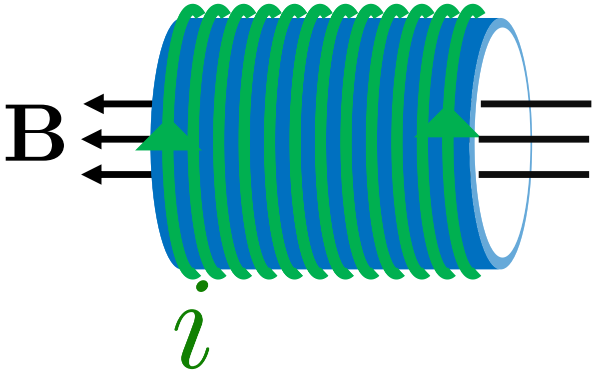
**B is the basic magnetic quantity!**

**Magnetic flux density *B* and Magnetic field strength *H***

# Chapter 4 The Critical Magnetic Field

**Ampere circuital Law**

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0 i$$



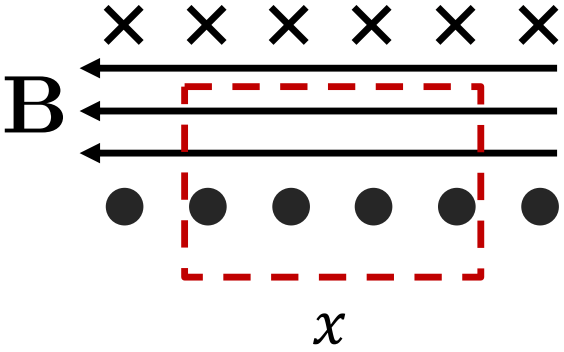
**Magnetic flux density inside an infinite long solenoid**

$$Bx = u_0 N i$$

$$\Rightarrow \boxed{B = u_0 m i}$$

$m = N/x$ turns per unit length

$mi : units(A/m)$

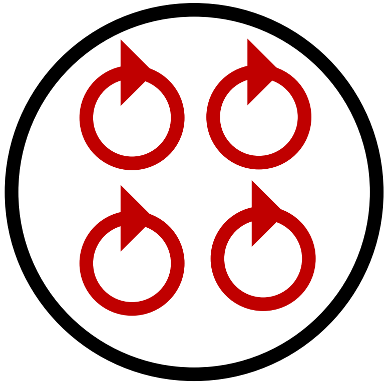


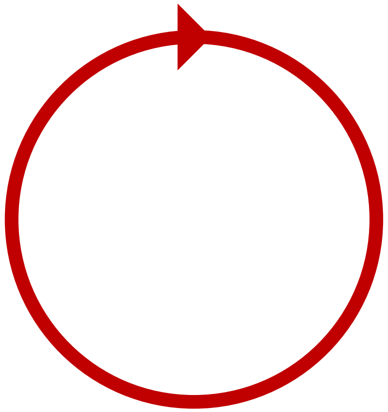
***N*** **turns enclosed by the closed red path**

**Magnetic flux density *B* and Magnetic field strength *H***

# Chapter 4 The Critical Magnetic Field

## Magnetization of a material



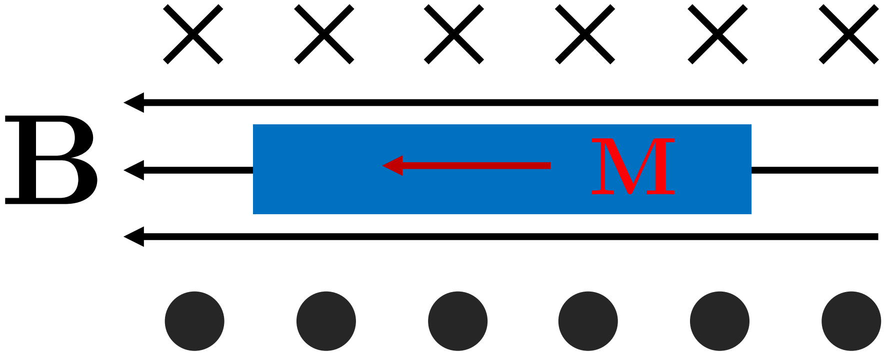


$$d\mathbf{u} = i\,d\mathbf{s}$$

units $A \cdot m^2$

$$\mathbf{u} = i \int d\mathbf{s}$$

**Magnetization** $$\mathrm{M} = \frac{\mathbf{u}}{V}$$ units $A \cdot m^{-1}$



**Flux density produced by magnetization**

$$\mathbf{B_m} = \mu_0 \mathbf{M}$$

**Total Flux density**

$$\boxed{\mathbf{B} = \mu_0 m \mathbf{i} + \mu_0 \mathbf{M}}$$

**Magnetic flux density *B* and Magnetic field strength *H***

# Chapter 4 The Critical Magnetic Field

$$\boxed{\mathbf{B} = \mu_0 m\mathbf{i} + \mu_0\mathbf{M}}$$

**Define a new vector**

$$\mathbf{H} = m\mathbf{i}$$

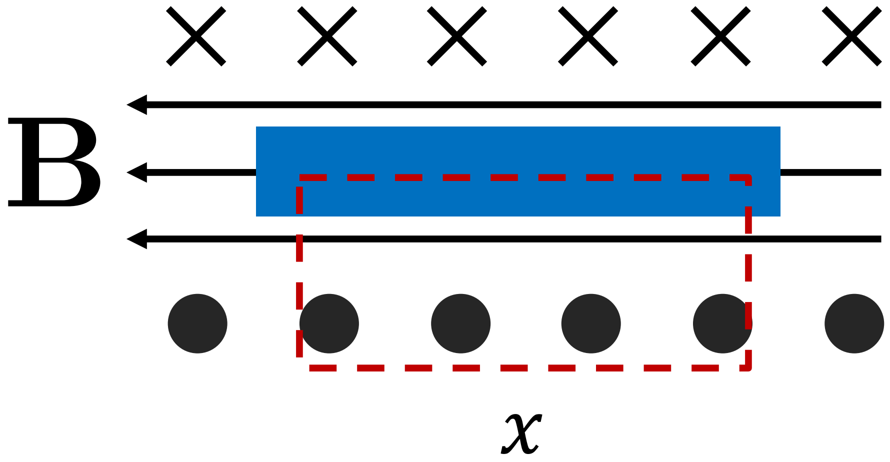
**The magnetic field strength**

**Does not depend on *M***

**Total Flux density**

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$$

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0(mix + Mx) = \mu_0(i_f + i_M)$$



$$\boxed{\oint \mathbf{H} \bullet d\mathbf{l} = \mu_0 mix = i_f}$$

$$i_f = xmi$$

**Free current**

**Magnetic flux density *B* and Magnetic field strength *H***

# Chapter 4 The Critical Magnetic Field

In small magnetic fields:

$$\mathbf{M} = \chi \mathbf{H}$$

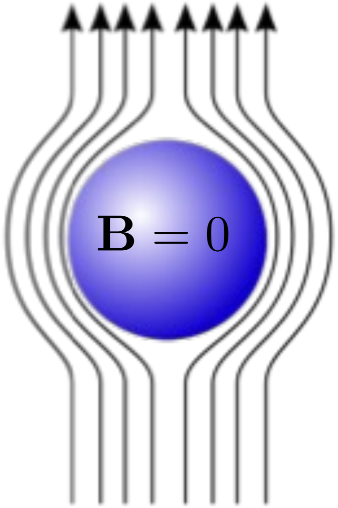
$\chi$ is the magnetic susceptibility

$$\Rightarrow \mathbf{B} = \mu_0(\mathbf{H} + M) = \mu_0(1 + \chi)\mathbf{H} = \mu_0\mu_r\mathbf{H}$$

$\mu_r = 1 + \chi$ is the relative permeability

For superconductors:

$$B = 0 \Rightarrow \begin{cases} M = -H \\ \chi = -1 \end{cases}$$



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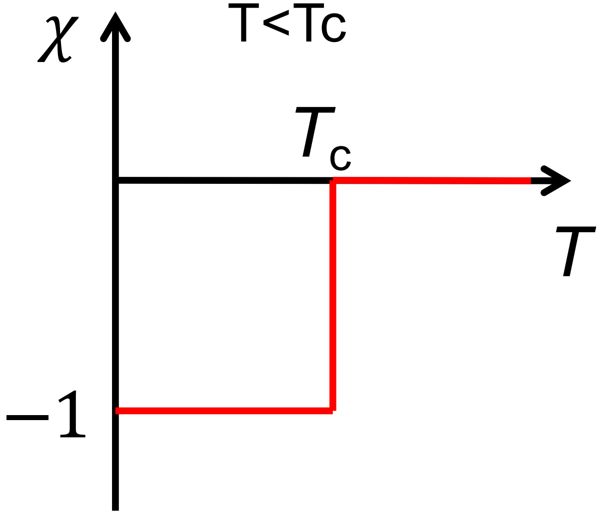


**Magnetic flux density *B* and Magnetic field strength *H***

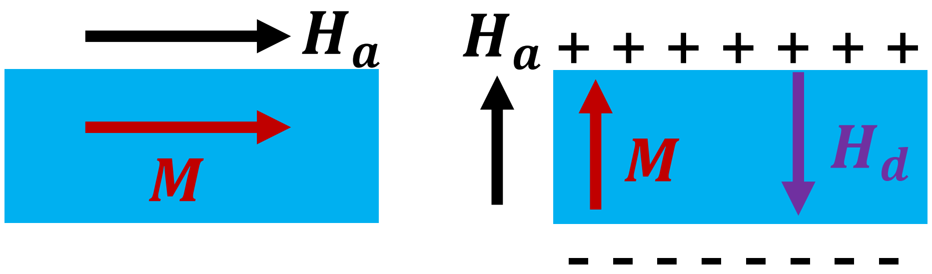
# Chapter 4 The Critical Magnetic Field

**Maxwell equation**

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0$$

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$$

- **H is not divergence free**
- **H behaves as if magnetic monopoles exist.**



**Demagnetization Effect**

For an ellipsoidal: $\mathbf{H}_d = -N\mathbf{M}$

The internal field and flux density are:

$$\begin{aligned} \mathbf{H}_i &= \mathbf{H}_a + \mathbf{H}_d \\ &= \mathbf{H}_a - N\mathbf{M} \\ &= m\mathbf{i} - N\mathbf{M} \end{aligned}$$

$$\begin{aligned} \mathbf{B} &= \mu_0(\mathbf{H}_i + \mathbf{M}) \\ &= \mu_0(\mathbf{H}_a - N\mathbf{M} + \mathbf{M}) \end{aligned}$$

For superconductor:

$$\mathbf{M} = -\mathbf{H} \qquad \mathbf{H}_i = \left(\frac{1}{1-N}\right)\mathbf{H}_a$$

**Demagnetization effect**

# Chapter 4 The Critical Magnetic Field

## Demagnetization Effect

In general

$$(H_d)_i = -\sum_j N_{ij} M_j$$

***M*** along the principal axes

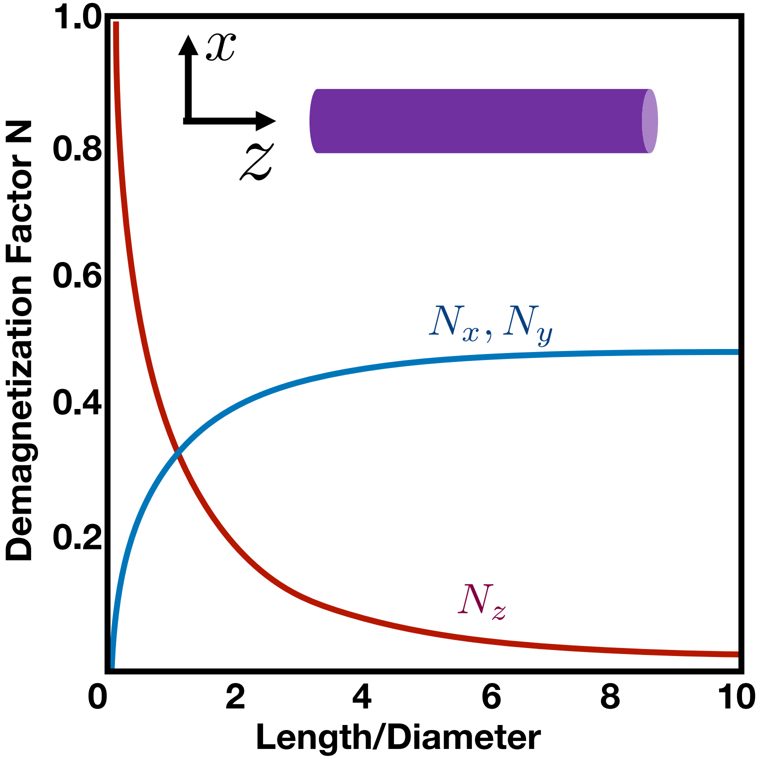
$$N = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$$

For a sphere

$$N_x = N_y = N_z = 1/3$$

For a long cylinder

$$N_x = N_y = 1/2, N_z = 0$$



**Magnetic flux density *B* and Magnetic field strength *H***

# Chapter 4 The Critical Magnetic Field

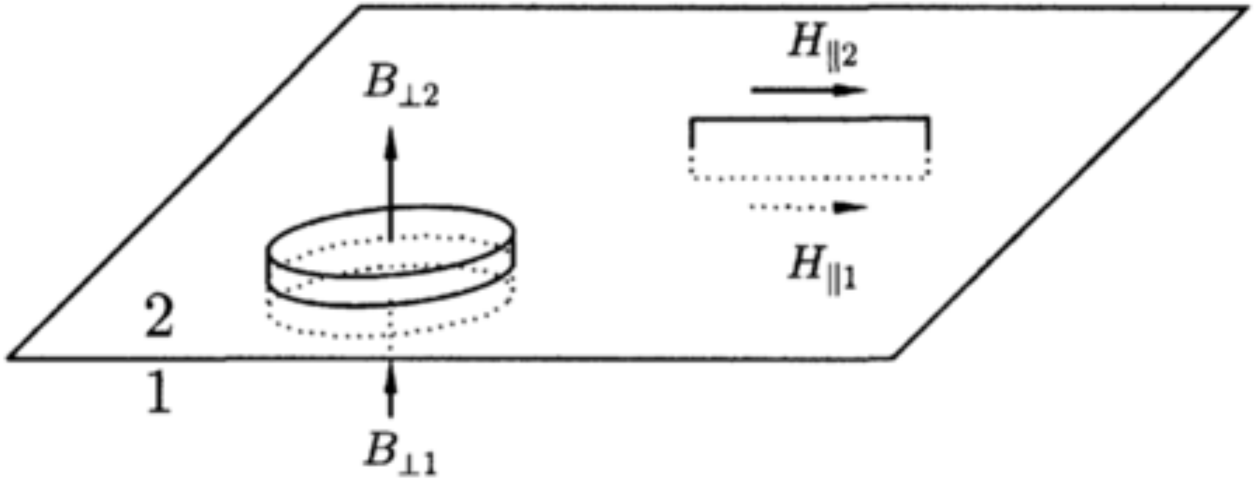
## Boundary Conditions

$$\nabla \cdot \mathbf{B} = 0 \rightarrow \int_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\Rightarrow B_{\perp 1} = B_{\perp 2}$$

$$\oint \mathbf{H} \cdot \mathbf{dl} = \int \mathbf{j_f} \cdot \mathbf{dS}$$

$$\Rightarrow H_{\parallel 1} = H_{\parallel 2}$$



Magnetic flux density *B* and Magnetic field strength *H*

# Chapter 4 The Critical Magnetic Field

Magnetic induction/Magnetic flux density (**B**) = Magnetic flux per unit area. Units: [Tesla = Weber/m$^2$ = Vs/m$^2$ = Kg/s$^2$/A]

Magnetization (**M**) = magnetic moment (**m**) per unit volume (V). Units: [A/m] $M = m/V$

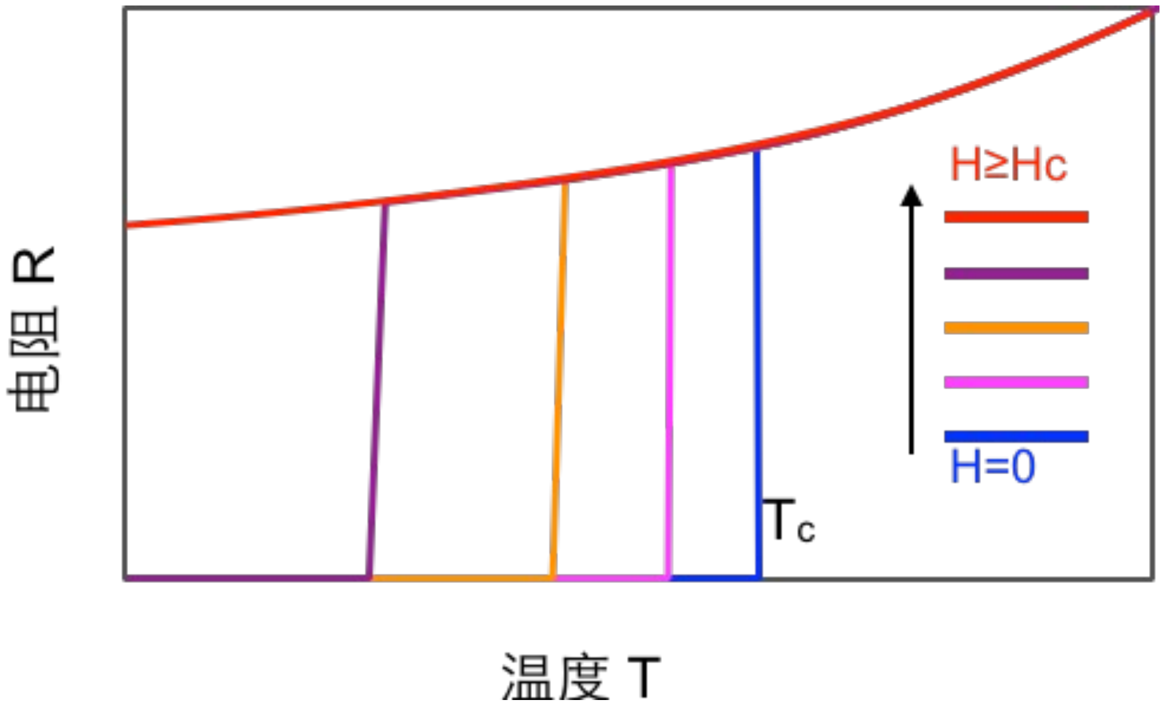
Magnetic field strength/Magnetizing force (**H**). Units: [A/m]

Magnetic susceptibility ($\chi$). (volume susceptibility) Units: [dimensionless] $\chi = M/H$

| Quantity | Symbol | Gaussian & cgs emu | Conversion factor, C | SI & rationalized mks |
|---|---|---|---|---|
| Magnetic flux density, magnetic induction | $B$ | gauss (G) | $10^{-4}$ | tesla (T), Wb/m$^2$ |
| Magnetic field strength, magnetizing force | $H$ | oersted (Oe), Gb/cm | $10^3/4\pi$ | A/m |
| (Volume) magnetization | $M$ | emu/cm$^3$ | $10^3$ | A/m |
| (Volume) magnetization | $4\pi M$ | G | $10^3/4\pi$ | A/m |
| (Volume) susceptibility | $\chi$ | dimensionless,<br>emu/cm$^3$ | $4\pi$ | dimensionless |

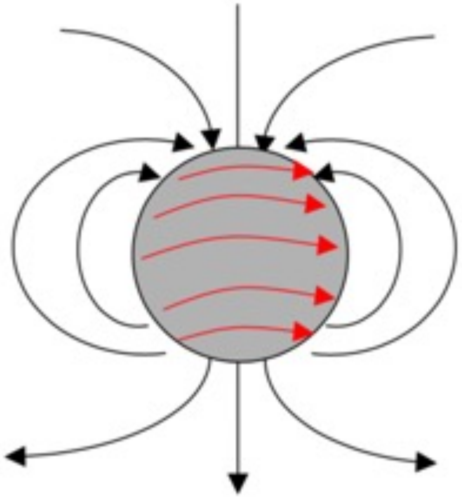
**Magnetic units**

# Chapter 4 The Critical Magnetic Field



$H_c$ Critical magnetic field strength

**Superconductor loose superconductivity when the applied magnetic field is larger than a certain critical value.**

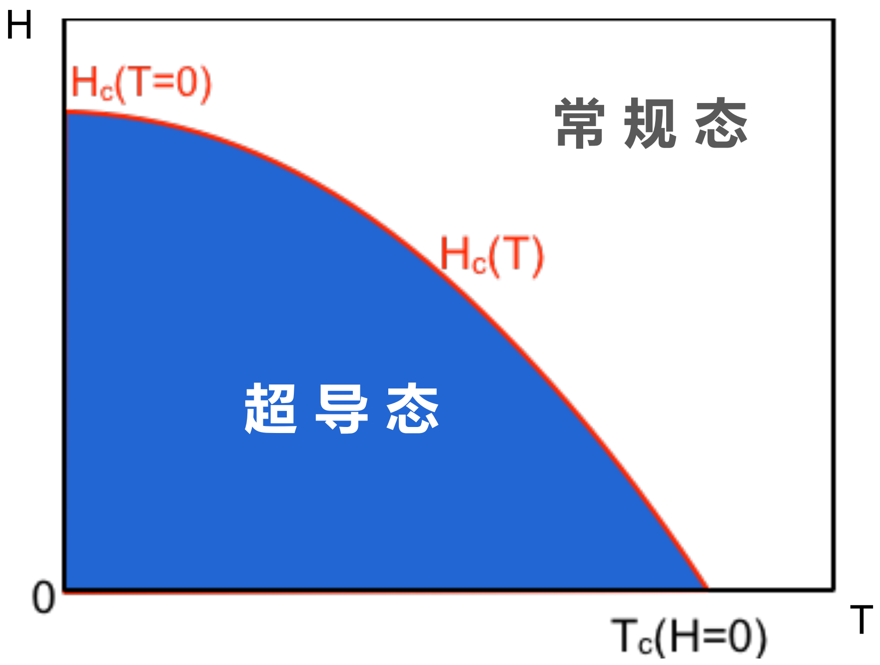


$$\nabla \times \mathbf{H_c} = \mathbf{J_c}$$

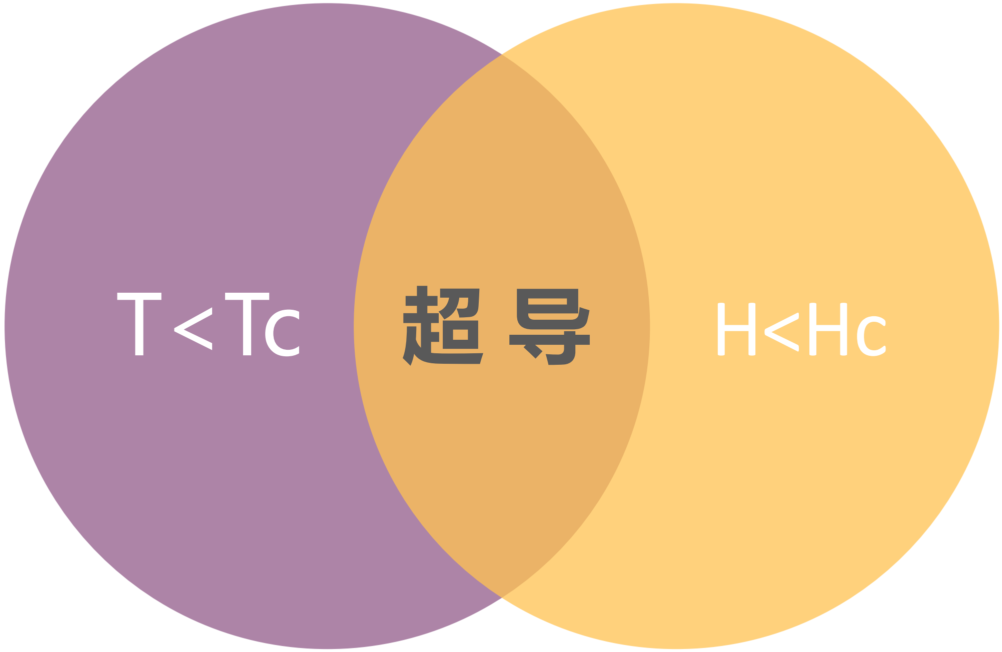
Critical current density $J_c$

**The Critical Magnetic Field**

# Chapter 4 The Critical Magnetic Field



$$H_c(T) \cong H_c(0)[1 - \left(\frac{T}{T_c}\right)^2]$$



**Phase diagram-相图**

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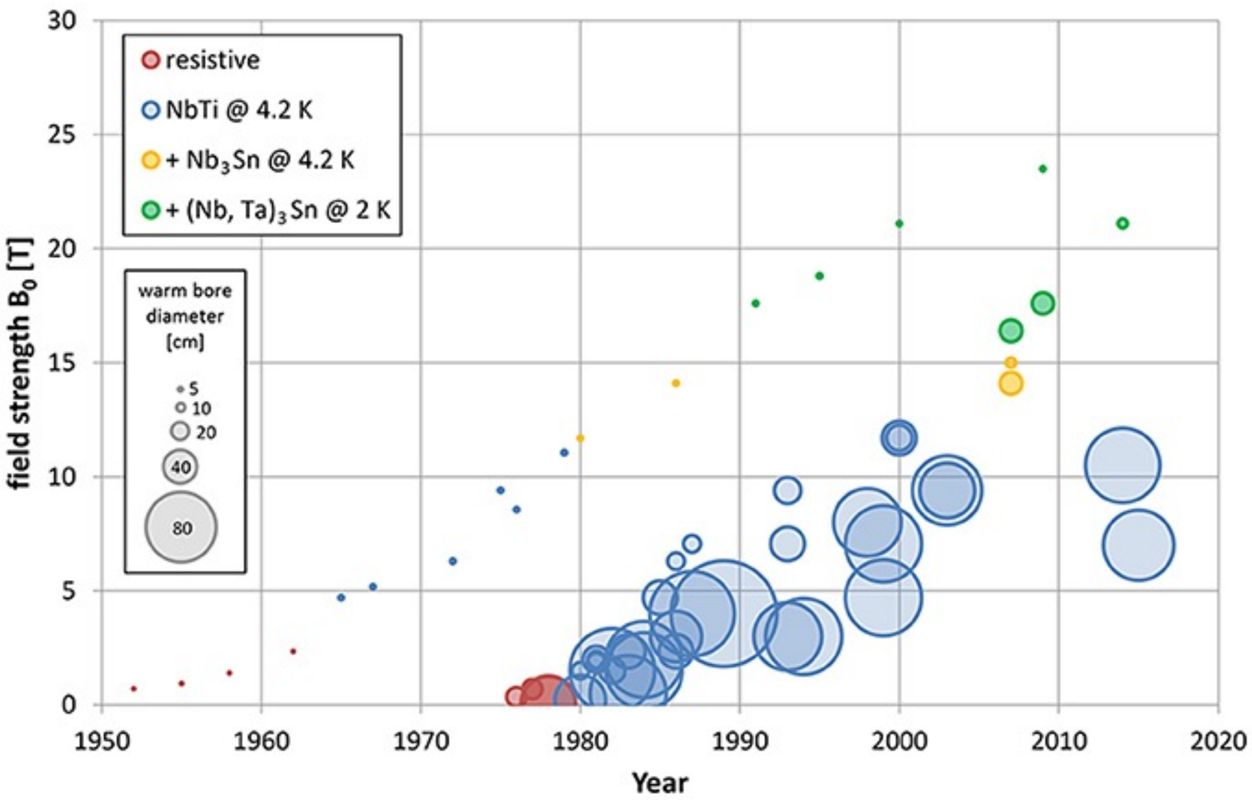

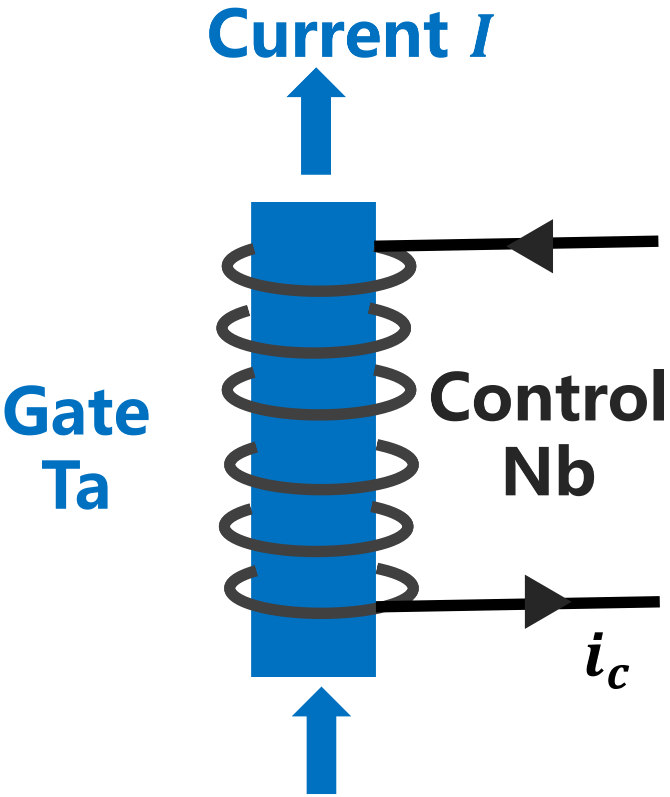

Current $I$

Gate Ta

Control Nb

$i_c$

**Application- Cryotron**

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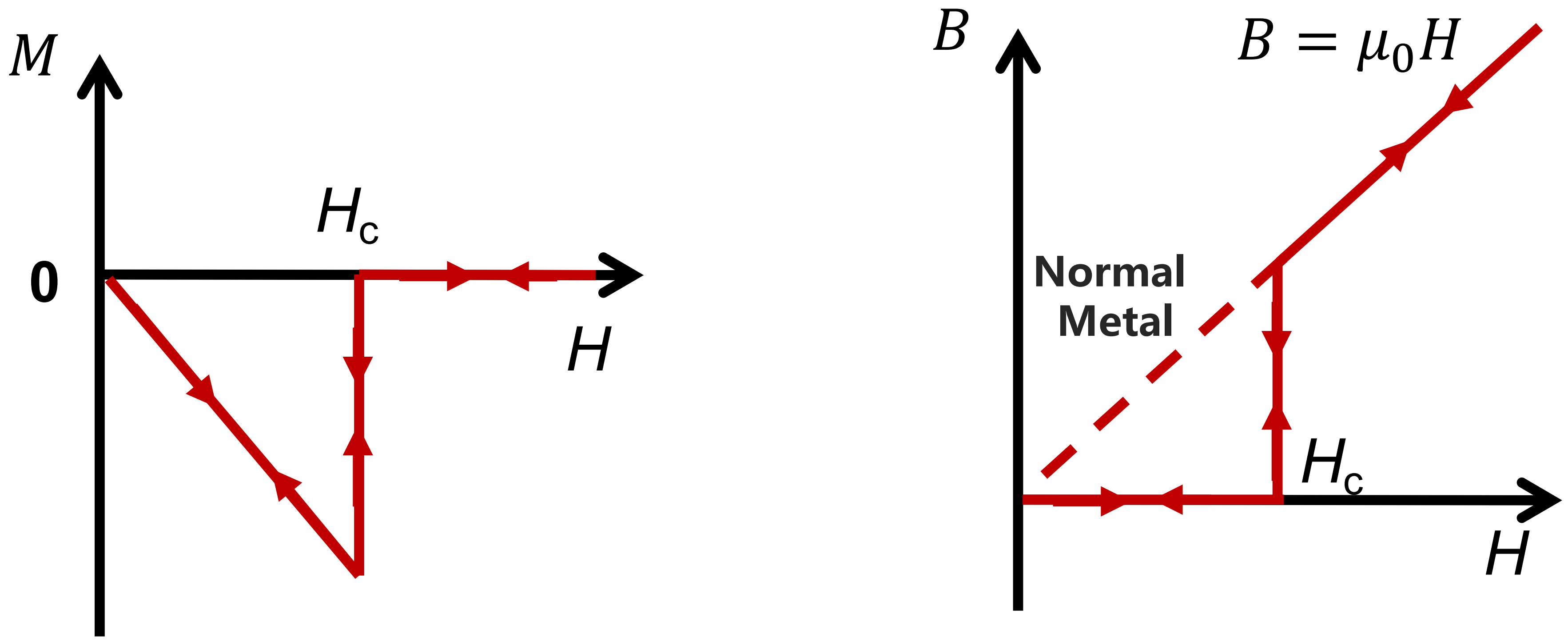

Reversible Magnetization for ideal specimens

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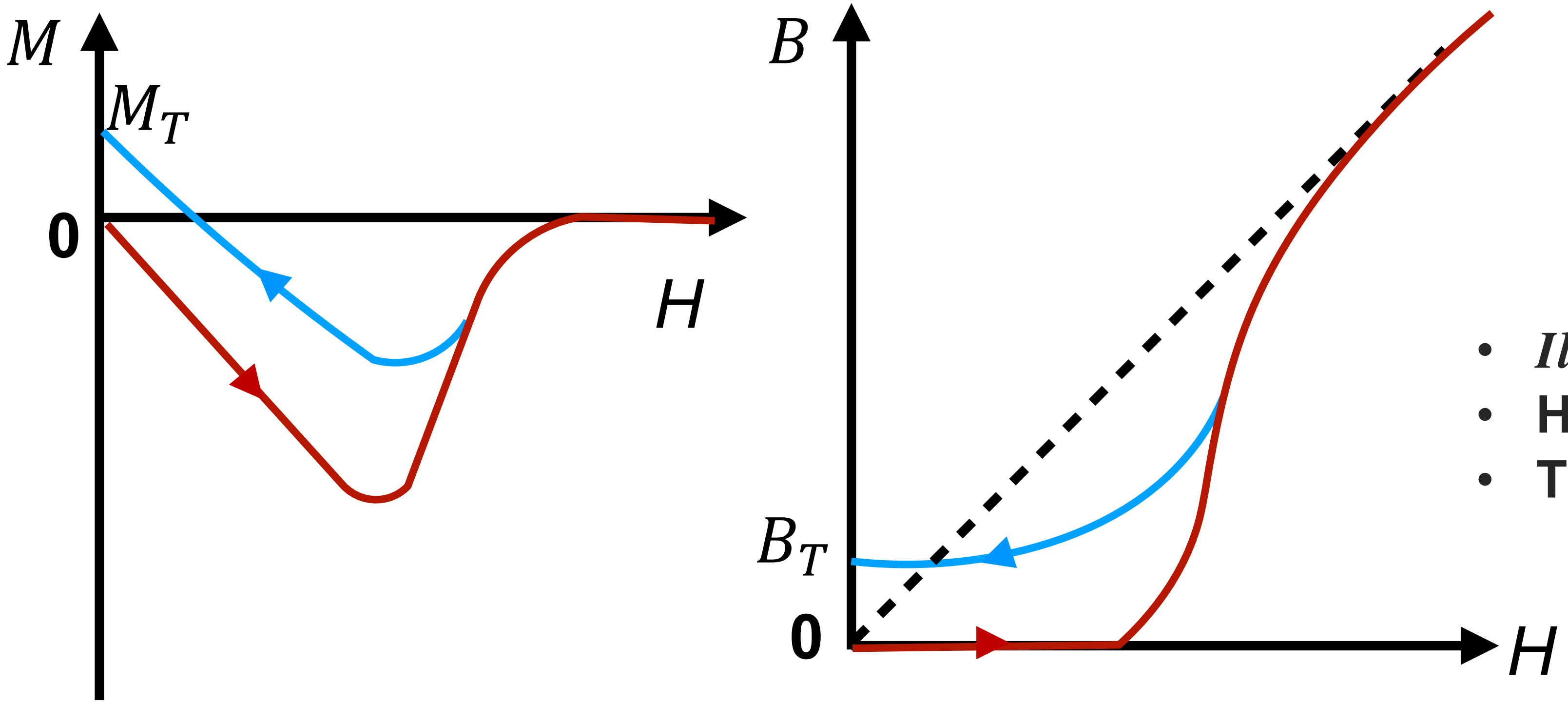

- *Ill* **defined Hc**
- **Hysteresis**
- **Trapped flux**

**Irreversible Magnetization for non-ideal specimens**

**Magnetization**

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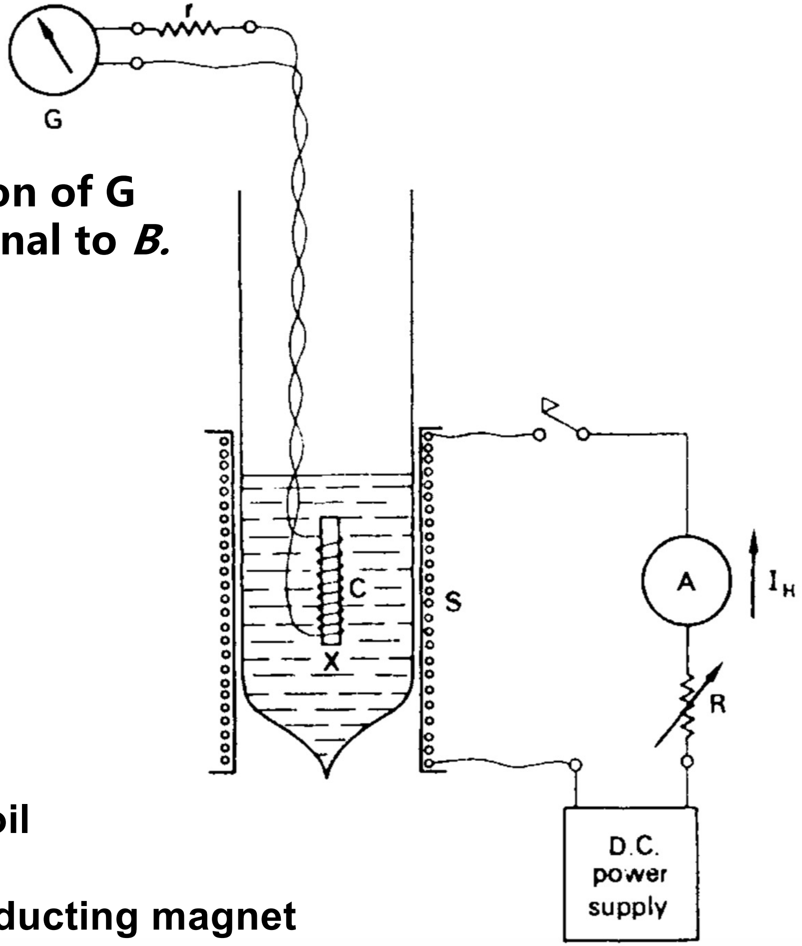
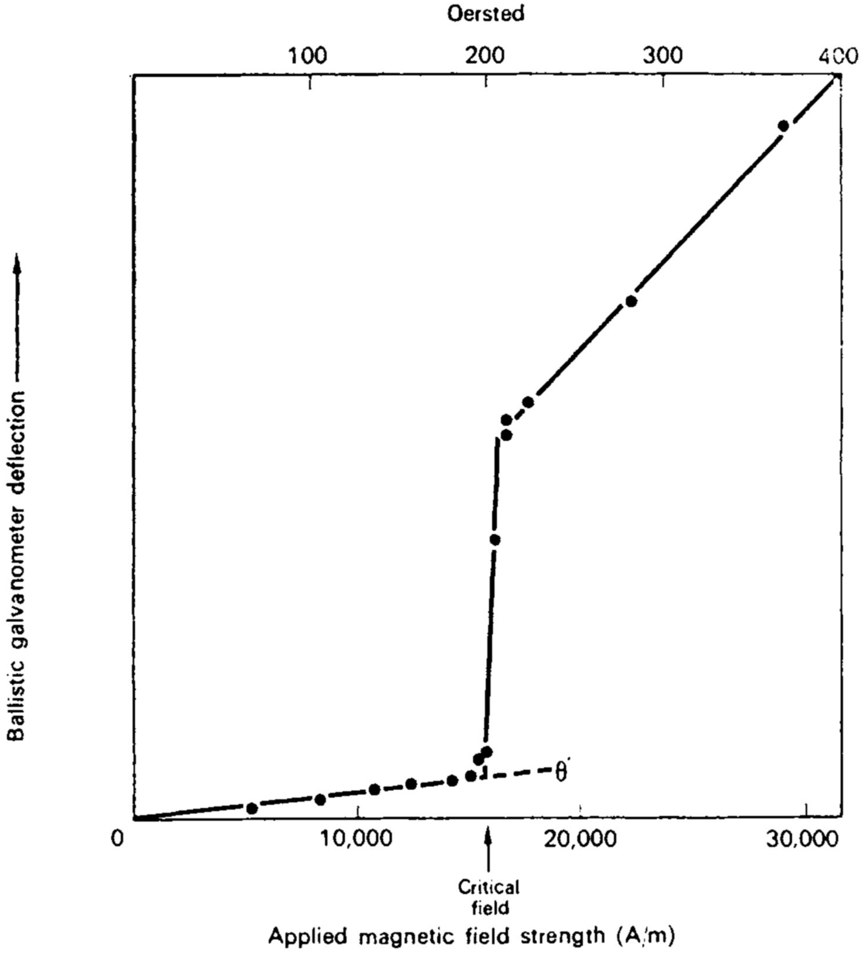
**The deflection of G is proportional to *B.***





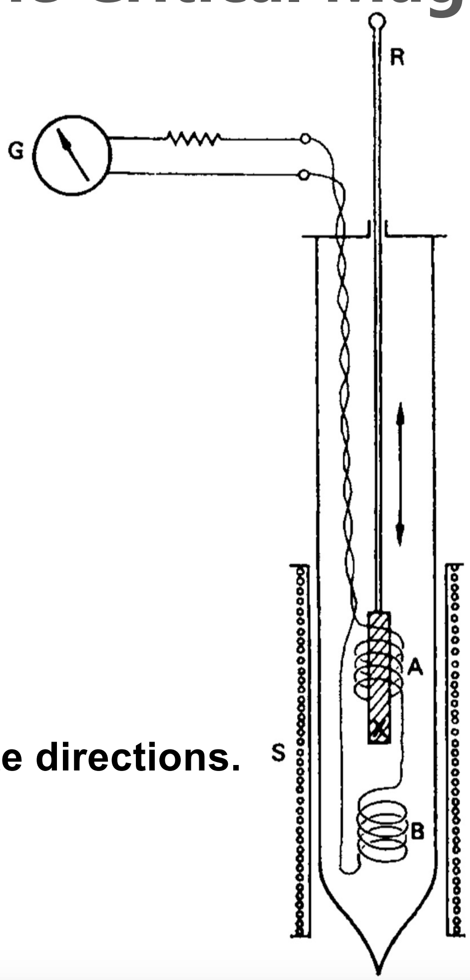
**C: pick up coil**
**X: Sample**
**S: Superconducting magnet**

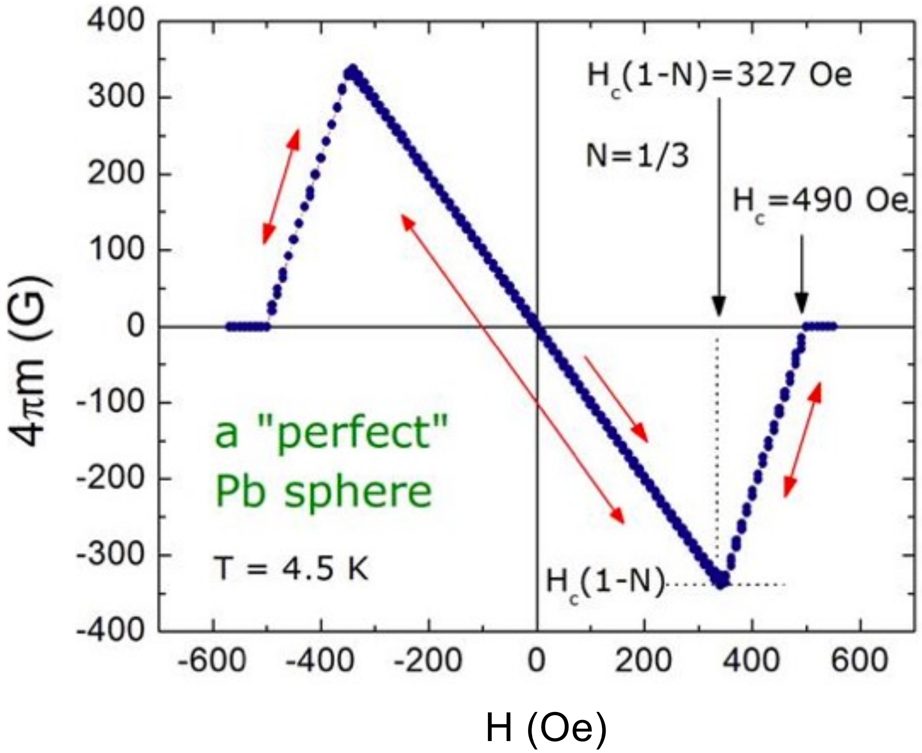
**Measurement of flux density**

# Chapter 4 The Critical Magnetic Field

**The deflection of G is proportional to *M.***

**Two pick up coils A and B.**
**Identical but wound in opposite directions.**





## Measurement of magnetization